Digital Filters

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“black box” view of a digital filter

- audio signal represented as a mathematical function $x(n)$
  - maps sample number $n$ to instantaneous amplitude

- input signal $x(n)$ process output $y(n)$
common applications

• modify the frequency spectrum of a signal
• apply frequency-dependent boost/cut
• emphasize or attenuate certain frequencies
• apply frequency-dependent phase changes
frequency spectrum

- representation of a signal in terms of sinusoids of specific amplitude, frequency and phase

- often graphed as frequency vs. amplitude (ignoring phase)

- the signal is no longer represented in terms of its evolution in time; it is represented in the *frequency domain*
example

- apply low pass filter (400 Hz cutoff, Q = 2.0)
sinusoids

\[ x(t) = A \sin(2\pi ft + \phi) \]

- \( A \) = amplitude
- \( \pi \) = 3.14159… half the circumference of the unit circle
- \( f \) = frequency in cycles per second (Hz)
- \( \phi \) = initial phase
significance of sinusoids

• objects that resonates or oscillate produces sinusoidal motion

• simple harmonic motion
  – consider a point that vibrates back and forth on the \( x \) axis
  – acceleration is proportional to position
  – differential equation

\[
\frac{d^2 x}{dt^2} = -a^2 x \quad \text{where } a \text{ is constant}
\]

  
  
  
  – satisfied by \( x = A \sin(at + \phi) \)
the human ear as spectrum analyzer

- the cochlea of the inner ear separates sound into into (quasi) sinusoidal components
- vibrations produce compression waves in cochlear fluid
- basilar membrane
  - 30,000 hair cells
  - frequency specific nerve endings
- positions on the basilar membrane correspond to sinusoidal frequency
implementing digital filters

• most practical digital filters are implemented in the time domain

• they look at the current and past values of the signal to determine the output
  – FIR – Finite Impulse Response

• they may also look at the current and past outputs of the filter (feedback process)
  – IIR – Infinite Impulse Response
digital filter equation

- for linear and causal digital filters

\[
y(n) = \sum_{i=0}^{M} a_i x(n - i) - \sum_{i=1}^{N} b_i y(n - i)
\]

- \(x(n)\) is the filter input
- \(y(n)\) is the filter output
- \(a_i\) and \(b_i\) the filter coefficients
digital filter design problem

• how do we determine the filter coefficients (time domain) in order to achieve a particular frequency response?

• given a set of filter coefficients, what is the frequency and phase response of the filter?

• use filter design algorithms
common filter types

- lowpass/highpass

- bandpass/bandstop

- peak/notch

- lowshelf/highshelf
filter parameters

- **frequency - in Hz**
  - center frequency (bandpass, notch, resonators, etc.)
  - cutoff frequency (lowpass, highpass, shelving, etc.)

- **bandwidth**
  - width in Hz from the +/-3dB boost/cut point

- **Q “quality factor”**
  - center frequency divided by bandwidth
  - as Q increases the filter bandwidth decreases
  - resonance increases
filters in Max/MSP

- `svf~`, `onepole~`, `reson~`, `ffb~`, `buffir~`

- `biquad~`

\[
\begin{align*}
y(n) &= \sum_{i=0}^{M} a_i x(n-i) - \sum_{i=1}^{N} b_i y(n-i) \\
    &= a_0 x(n) + a_1 x(n-1) + a_2 x(n-2) - b_1 y(n-1) - b_2 y(n-2)
\end{align*}
\]

- use `filtergraph` object to generate the 5 coefficients for `biquad~`
End
sine and cosine

- $\sin(\theta)$
  - projection of the y component of a point on the unit circle rotated at angle $\theta$

- $\cos(\theta)$
  - projection of the y component of a point on the unit circle rotated at angle $\theta$